Journal of Structural Geology, Vol. 10, No. 5, pp. 517 to 527, 1988
Printed in Great Britain

0191-8141/88 $03.00+0.00

Strain refraction in layered systems

SUSAN H. TREAGUS

Department of Geology, The University, Manchester M13 9PL, U.K.

(*Received* 24 *July* 1987; *accepted in revised form* 7 *March* 1988)

Abstract—Strain refraction across competence contrasts is presented as a simple model consisting of two components, a homogeneous strain and a heterogeneous simple shear. For Newtonian materials, the ratio of the layer-parallel simple shear component in adjacent layers is the inverse of their viscosity ratio. Strong changes in ellipsoid size, shape and orientation are predicted across viscosity contrasts.

The geological implications of strain refraction theory are considered within the context of the 'cleavage/strain debate'. The particular relationships of relative competence and strain revealed by the refraction model may contribute to the problem of why cleavages of different morphologies in rocks of different lithologies (and kinematic histories) should appear to be subparallel to the XY planes of measured strain ellipsoids. Competent rocks should develop dominantly layer-orthogonal strain, and incompetent layers shear-dominated deformation. A variety of structural features ranging from cleavage refraction, changing lineation orientations, folds transected by cleavage, changes from coaxial to non-coaxial deformation, and ramp-flat fault geometry may be the result of stress and strain refraction in rocks.

INTRODUCTION

THE TERM *strain refraction* is used in this paper to describe changes in strain orientation and intensity across a planar boundary between two different materials. It has been shown that, in theory, stresses oblique to layering in multilayered viscous continua should refract across a viscosity contrast (Strömgård 1973, Treagus 1973, 1981). Following this theory, finite-strain refraction was modelled theoretically for planar Newtonian layers in perfect adherence (Treagus 1983).

It was shown in Treagus (1983) that homogeneous strain should only exist in multilayers with viscosity contrasts, if the principal strains are layer-parallel and layer-orthogonal. In all other cases, principal strain axes will refract from layer to layer, and their magnitudes vary sympathetically. Using the strain axis convention, $X \geqslant Y \geqslant Z$, the XY plane was found to refract towards layer-normal, with a decrease in strain, in more viscous layers, and toward the layer, with an increase in strain, in less viscous layers (Fig. 1). In these 2D models, two axes refract and the third is constant in the plane of layering. Thus the strain ellipsoids refract on a common principal axis. For the case of plane strain with $Y = 1$ parallel to layering (Fig. 1), strain refraction is truly 2D. For other cases ($Y \neq 1$, or X or Z parallel to layering), ellipsoid variations are not fully appreciated in a 2D view. Figure 2 is an example in which refraction of strain axes is associated with such marked changes in axial values that a changeover of principal axes (X and Y) occurs.

Treagus (1983) compared theoretical strain refraction patterns to sense of *cleavage refraction* across different lithologies, as described by Sorby (1853) and Harker (1886). However, her review of the evidence for assuming that refracted cleavage exactly tracks refracting XY planes in different lithologies was inconclusive. The main proof that cleavage is parallel to XY planes comes from well-cleaved slates (Siddans 1972, Wood 1974) and comparisons of cleavage patterns to theoretical strain patterns in folded layers (Dieterich 1969).

Ramsay has illustrated patterns of strain refraction across competence contrasts (Ramsay 1982, Ramsay & Huber 1983, p. 184, 1987, p. 462) that are qualitatively similar to the theoretical patterns in Treagus (1983). Strain refraction, here, is modelled on cleavage refraction: "cleavage planes represent the trace of XY planes of adjacent strain ellipsoids and . . . cleavage must obey the geometric rules of finite-strain trajectories" (Ramsay & Huber, p. 184). Cleavage refraction towards bedding is indicative of layers of lower competence, and thus an 'order of competence' can be determined for a sequence of rock types (Ramsay 1982).

The relationship between strain and cleavage and thus between strain refraction and cleavage refraction is important and still not fully understood. A later section of the paper is devoted to this topic.

In Ramsay's model, *competence* is equated with *ductility*, and is essentially a measure of strain intensity. However, Hobbs *et al.* (1976, p. 67) distinguished competence, a measure of relative strength, from ductility, the ability to undergo permanent strain. In theoretical analyses of buckling layers, the 'competent layer' has been used to mean either a greater modulus of elasticity or a greater viscosity (Ramberg 1964, Hara & Shimamoto 1984, p. 194). Lister & Williams (1983) suggested an alternative definition for competence. They considered deformation in rock masses as partitioned into zones with different flow histories, where competent layers are characterized by coaxial deformation and incompetent layers non-coaxial deformation. Clearly, competence is not equatable to any single physical prop-

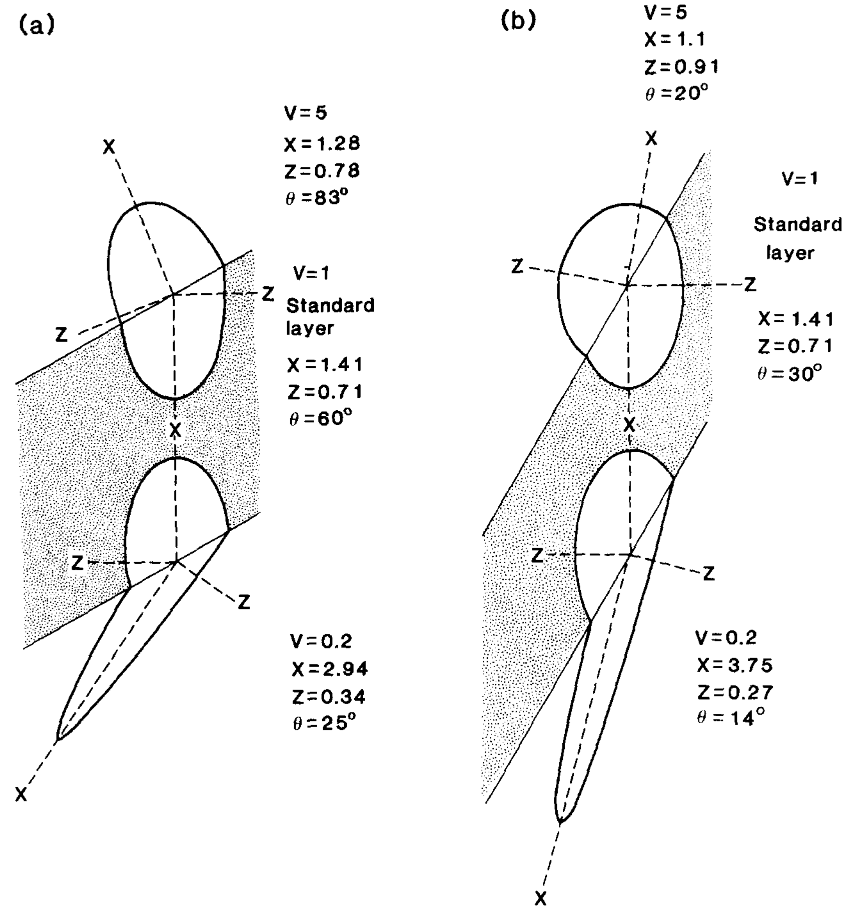

Fig. 1. Two examples of 2D finite-strain refraction across planar layers with Newtonian properties, after Treagus (1983, fig. 6 and table 2). The stippled layers show the 'standard' strain ellipse (axial ratio 2), oriented with X at 60° to layering in (a), and 30° in (b). Strain refraction is shown for a layer with viscosity ratio $V = 5$ ($5 \times$ standard) above, and $V = 0.2$ below. $Y = 1$ perpendicular to page, throughout.

erty and it may, according to circumstances, involve relative ductility, viscosity, or strength. In the present paper, rock behaviour is modelled as Newtonian viscous, and *competence contrasts* are used to describe *effective viscosity ratios* in rocks.

Ramsay & Huber (1983, p. 184) suggested that cleavage/strain refraction should be considered in terms of three independently compatible components of strain analogous to the strain components in ductile shear zones (Ramsay & Graham 1970, Cobbold 1977, Ramsay 1980): (1) homogeneous strain; (2) heterogeneous simple shear; and (3) heterogeneous dilation perpendicular to layering. The first two components are the basis of the theoretical model of strain refraction in Treagus (1983) and this paper. The third component is not included in this analysis, because layers are modelled as incompressible.

The present strain refraction model considers a system of *planar layers* in which stresses and strains refract from one homogeneous state to another in accordance with

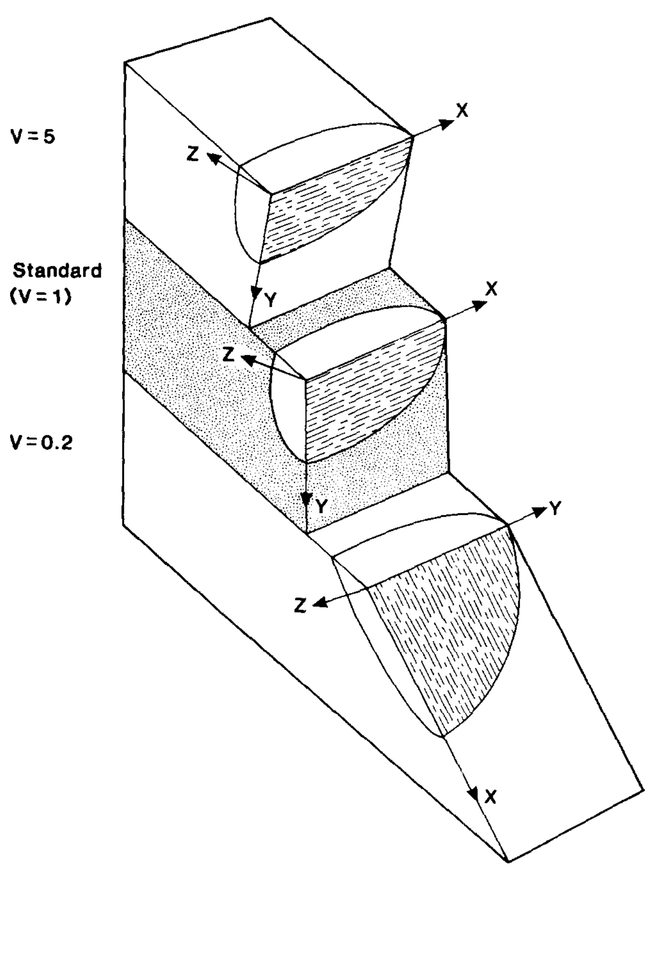

Fig. 2. Schematic representation of 3D shape variation of ellipsoids refracting in 2D. X is parallel to layering in the standard layer (stippled) and is the axis of refraction; this becomes the Y axis in the $V = 0.2$ (less competent) layer, because of the increased layer-parallel shear component. The refracted XY planes are shaded with lines parallel to X. Note that all ellipsoids have the same layer-parallel sectional ellipse.

viscosity changes. The refraction of strain axes is necessary for any multilayer with viscosity contrasts which is in *oblique strain*, either in 2D or 3D. Treagus (1983) proposed that it was the obliquity of straining which gave rise to strain and cleavage refraction on fold limbs, rather than a particular mechanism of folding. However, Ramsay (1967, p. 403), Henderson *et al.* (1986) and Ramsay & Huber (1987, p. 462) described cleavage refraction and cleavage fanning in terms of idealized models of strain in folds: tangential longitudinal strain and flexural slip (layer-parallel simple shear). In this case, all strain refraction is fold-related.

Layered systems of different competence, such as are modelled here, would be expected to buckle in compression and develop the kinds of fold-related strain variations computed from finite-element modelling (e.g. Dieterich 1969, Shimamoto & Hara 1976, Hara & Shimamoto 1984). I consider such strain variations to be two-fold: (1) *strain fans*, which are patterns of inhomogeneous strain associated with bending in and around buckle-fold hinges (see Roberts 1971, Cobbold & Barbotin 1988), and (2) *strain refraction*, which involves changes of homogeneous strain across layer competence contrasts, as seen in fold limb zones. The former considers kinematically compatible inhomogeneous strains and strain gradients (e.g. Cutler & Cobbold 1985, Cobbold & Barbotin 1988, and source references therein) within one materially homogeneous region. The latter considers the mechanics and kinematics across materially different layers, with a planar interface (Treagus 1981, 1983, Cobbold 1983). Both models would seem equally important in the investigation of strain in folded multilayers, because natural strain patterns must obey the rules of both together. These combined rules will determine the mechanism of folding.

The two components of strain refraction considered in this paper are layer-parallel/orthogonal strain and layer-parallel simple shear. These are equivalent to Ramsay's (1967, pp. 398 & 391) ideal fold-strain models, tangential longitudinal strain and flexural slip. Refraction modelling thus provides the mechanics for apparent changes in folding 'mechanism' across competence contrasts, and offers the means to quantify these components in terms of multilayer viscosity ratios.

MODEL OF STRAIN REFRACTION ACROSS VISCOUS LAYERS

The theoretical model of strain refraction is taken from Treagus (1973, 1981, 1983). Layers are assumed planar, semi-infinite, Newtonian viscous, isotropic and incompressible. Given a known state of homogeneous strain in one layer, with principal axes oblique to layering, can we determine the state of strain in an adjacent layer of known viscosity ratio? This may be called the *refracted strain*. The solution depends on two 'rules' which are given here in the general 3D form for strain ellipsoid refraction.

(1) *The refracted ellipsoid shares the same ellipse section with the known ellipsoid at the layer interface*. This defines the geometrical compatibility condition for coherent layers.

(2) *The shear stress on the interface plane will be equal in adjoining layers*. This defines a condition of mechanical compatibility for adherent layers. Equal layer-parallel shear stresses in adjacent layers of different viscosity must, by definition, signify unequal shear strain rates, which 'jump' in inverse proportion to the viscosity. An important simple relationship exists for finite layer-parallel shear strains across a planar viscosity boundary in Newtonian materials (see Cobbold 1983, Treagus 1983, appendix 1): *the finite (and incremental) shear strain ratio across the interface is the inverse of the viscosity ratio*. (This is not the case for non-linear rheologies whose viscosities are a function of strain, see Cobbold 1983, unless they changed by the same factor so that their viscosity ratios stayed constant.)

The two rules above are the basis of the algebraic and graphical solutions for 2D strain refraction presented in Treagus (1983). They are being applied currently to deriving solutions for 3D refraction. No 3D algebraic solutions have been found, comparable to those for 2D in Treagus (1983, table 1). Results to date have been derived by graphical methods, using 'Mohr loci' on the Mohr diagram for 3D strain (Treagus 1986).

Any system of refracting strain ellipsoids can be considered to comprise two finite components (Fig. 3).

Component (a) is a homogeneous state of layer-parallel/perpendicular strain that distorts the whole multilayered system equally. This satisfies rule (1) above, and is abbreviated, here, to the *layer-orthogonal strain component*. The layer-parallel strain may be a shortening or an extension.

Component (b) is a heterogeneous state of layer-parallel simple shear that distorts the system such that across each viscosity interface, the layer-parallel shear strain is multiplied by the inverse viscosity ratio. This satisfies rule (2). By definition, this component is plane strain.

For 2D models, the separation of strain refraction into components of homogeneous strain and heterogeneous simple shear is easy to visualize and to illustrate, as shown in Fig. 3. The ellipsoids for both components share a principal axis parallel to the interface (layering), which is the axis of refraction of the finite-strain ellipsoid. Mohr circles can be used to illustrate the relationships between the two components (Fig. 4).

For the general 3D case, the two components do not have a common principal strain axis in the layering plane, but are mutually oblique (Fig. 5a). The combined effects of (a) homogeneous strain and (b) oblique heterogeneous simple shear result in finite-strain ellipsoids which refract in 3D (Fig. 5b). There is, therefore, no unique cross-section which can illustrate 3D strain refraction in 2D.

It must be emphasized that the two components (a and b) operate together incrementally. Nevertheless, it is convenient to model the finite-strain ellipsoids as a finite component of homogeneous layer-orthogonal strain (a) in all layers, followed by finite layer-wide simple shear

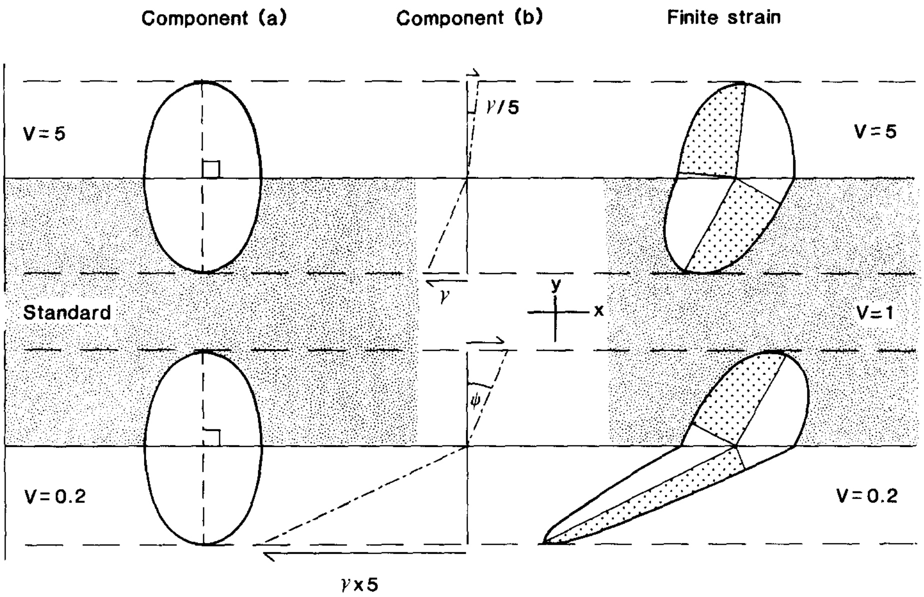

Fig. 3. The 2D finite-strain refraction example in Fig. 1(a) can be modelled as a homogeneous strain component (a) followed by a heterogeneous simple shear component (b) which differentially shears component (a) as a 'card-deck' to produce the finite refracted ellipses. The shear component refracts as the inverse viscosity ratio. Standard layer stippled.

zones (b) with shear strain inversely proportional to viscosity ratio (Fig. 6). Component (b) thus distorts (a) in card-deck fashion. An alternative model of heterogeneous simple shear followed by layer-orthogonal homogeneous strain would be equally valid, but is less convenient because the values of simple shear would not now be equivalent to the layer-parallel shear strains, but would include a factor of the homogeneous strain component (see Sanderson 1982).

By factorizing refracted strain ellipses and ellipsoids into homogeneous and heterogeneous components, these strain patterns become comparable to those in many other studies of heterogeneous strain patterns. The strain variations in a multilayer might thus be considered to represent a system of ductile shear zones (Ramsay & Graham 1970) on the layer scale, or equivalently, a form of banded deformation (Cobbold 1977, Cutler & Cobbold 1985), where the bands are the layers, and each is a compatible homogeneous strain domain (Means 1983). Alternatively, the whole multilayer deformation can be regarded as partitioned into different flow histories in different layers, as described by Lister & Williams (1983). The factorization of strain in thrust belts into components of pure shear and simple shear (Coward & Kim 1981, Sanderson 1982) has some similarities with factorization of strain refraction. However, these methods are concerned with patterns of regional strain variation, rather than strain compatibility on a domainal scale or variations across contrasting layers.

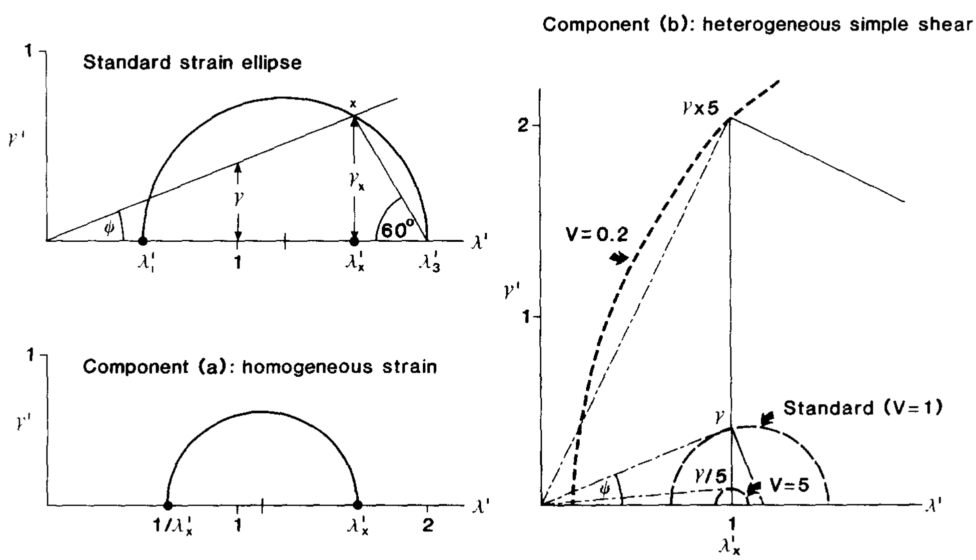

Fig. 4. Mohr circles of the components of plane strain refraction in Fig. 3. The homogeneous layer-orthogonal component (a) is given by λ'_x, and $1/\lambda'_x$ from the standard ellipse. The simple shear components (b) are given by γ for the standard ellipse ($V = 1$) and γ/V for the refracted ellipses ($V = 5$ and 0.2).

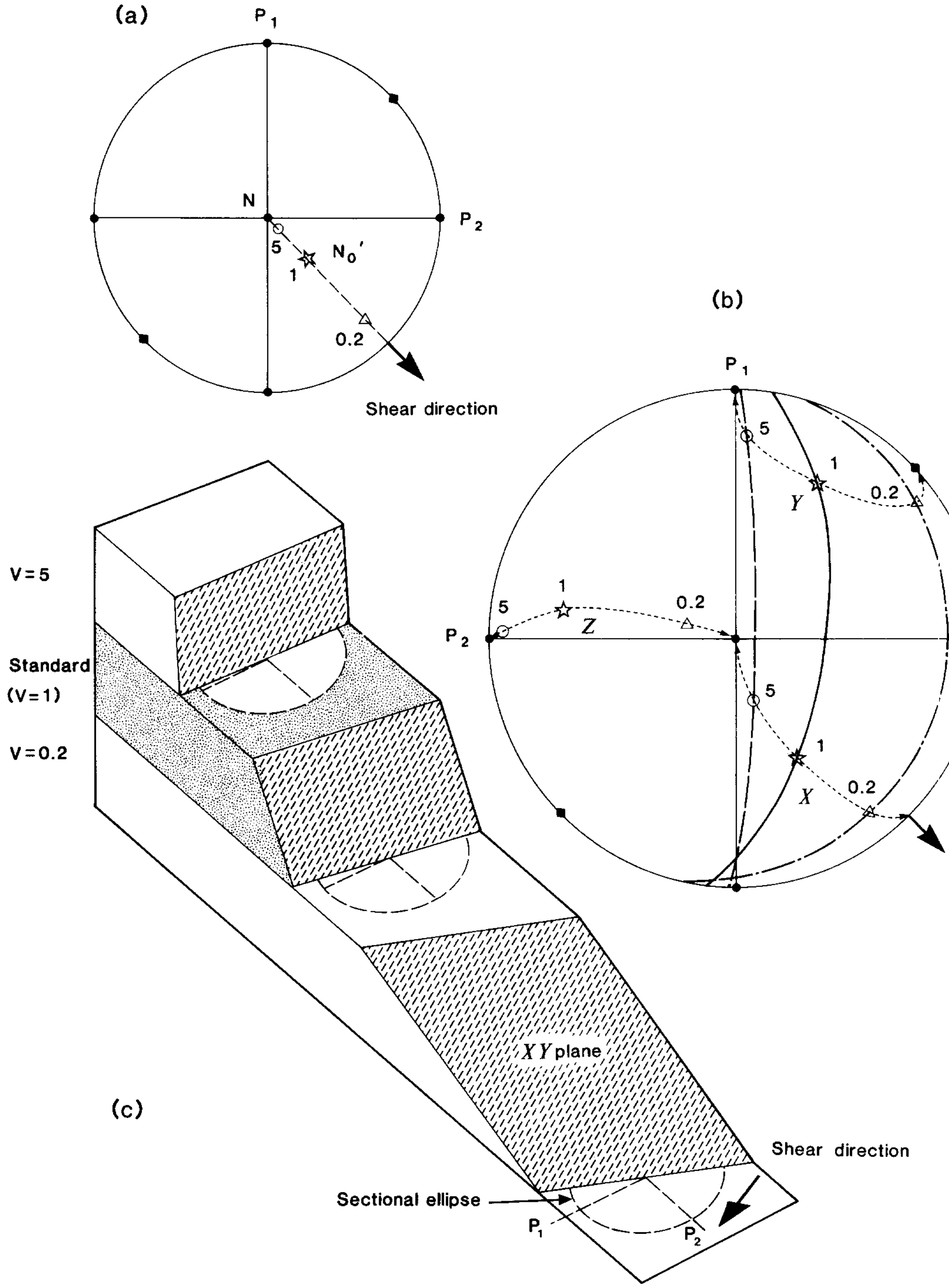

Fig. 5. An example of 3D strain refraction. (a) and (b) Lower-hemisphere equal-area projections with layering horizontal. (a) Strain components: homogeneous layer-orthogonal component on axes P_1, P_2, N, in all layers; heterogeneous layer-parallel simple shear components in direction of arrow, values given by sheared layer-normals (N_0') for 'standard' (V = 1, star), V = 5 (circle) and V = 0.2 (triangle) layers. Diamond at 90° to shear direction, in layering. (b) Orientation of finite-strain axes (X, Y, Z) for V = 1 (stars), V = 5 (circles) and V = 0.2 (triangles) layers. XY planes shown by solid curve for V = 1, broken for V = 5, and dot-dash for V = 0.2; their differences in strike demonstrate their different intersections with layering. End member refraction trends are shown by fine broken curves, with small arrows indicating positions of X, Y, Z for $V \rightarrow \infty$ and $V \rightarrow 0$. (c) Schematic 3D drawing of the strain refraction in (b). Standard layer stippled. XY planes shaded with lines parallel to X, for each layer. Note that all layers have the same sectional ellipse (P_1, P_2) and direction of shear, but different intersections of XY planes with layering.

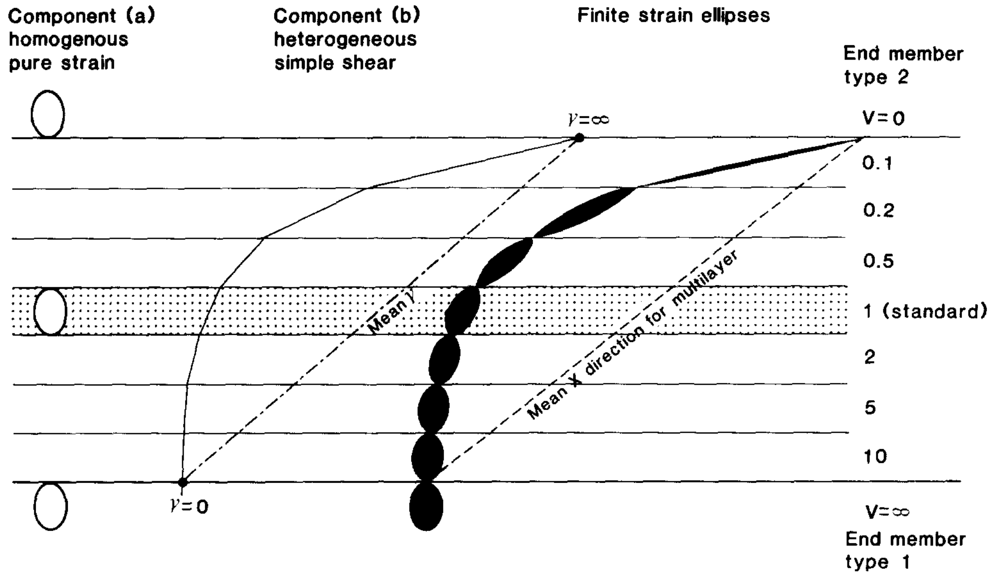

Fig. 6. Finite-strain ellipse refraction (black ellipses) across a set of layers with a range of viscosity ratios to the stippled standard layer. The homogeneous layer-orthogonal strain is shown on the left, and the heterogeneous simple shear given by the γ trajectory of the original bedding-normal. The mean X trajectory is almost parallel to the mean γ trajectory, but at a slightly smaller angle to layering. Note that the standard layer ($V = 1$) does not represent the average strain for this system. The end members of strain refraction are layer-parallel strain (zero γ) for $V = \infty$, and infinite layer-parallel simple shear + layer-orthogonal strain for $V = 0$.

FEATURES OF STRAIN REFRACTION MODELLING

Many of the features of strain refraction have been described in detail in Treagus (1983); others were predicted in Treagus (1981). The present model of strain refraction in terms of a homogeneous component of layer-orthogonal strain and a heterogeneous component of layer-parallel simple shear allows these features to be more easily understood.

For any analysis of strain refraction it is assumed that there is a known state of strain in a particular layer, which becomes the *standard layer*. Strain states in adjacent layers are described in terms of their viscosity ratios to the standard layer. Although it would be convenient to consider the standard layer as representing the 'bulk' behaviour and the *bulk strain* in a multilayer, this would not be correct. Figure 6 demonstrates that no single layer in the seven-layer multilayer represents the 'bulk' or average strain. The bulk shear strain is given by the arithmetic means of all γs, and thus the bulk (shear) viscosity ratio is the reciprocal mean of all Vs (0.37 in Fig. 6). But the bulk behaviour is anisotropic, whereas all individual layers are isotropic (see Biot 1965, p. 186, Cobbold *et al.* 1971). The 'standard' layer simply has the median viscosity of the system.

The main features of finite-strain refraction are summarized for three orientations of standard and/or bulk strain.

Layer parallel/orthogonal finite strain

If the standard strain ellipsoid has principal axes parallel and perpendicular to layering (whether layer-parallel compression or extension), there will be no strain refraction because there is no component of layer-parallel shear strain. Therefore, despite viscosity contrasts, there will be *finite homogeneous strain*.

Two-dimensionally oblique

Strain ellipsoids which have one principal axis parallel to layering will refract about this common principal axis. Strain refraction can be viewed in 2D by patterns of ellipse refraction (e.g. Figs. 1, 3 and 6). For the case of plane strain in the standard layer ($Y = 1$ parallel to layering), refracted strain ellipsoids will also be plane strain. In all other cases variations of ellipsoid shape will arise from refraction, and in certain circumstances, the principal axis in layering (the refraction axis) may change from X to Y, or Y to Z (Treagus 1983) (e.g. Fig. 2).

The simplest way to analyse strain refraction patterns in 2D is to distinguish components (a) and (b) in the standard layer (Fig. 6). Then for example, a layer with viscosity ratio $V = 10$ (10 × standard) has the same component (a) and 0.1 × component (b). Its refracted ellipsoid is thus very close to component (a), which in this 2D case is layer-orthogonal coaxial strain (pure shearing). A $V = 0.1$ layer also has the same component (a), but 10 × component (b). Its finite strain will be dominated by the simple shearing component but will not be perfect layer-parallel simple shear unless component (a) is zero (i.e. the case of heterogeneous simple shear).

Three-dimensionally oblique

This is the general case where ellipsoids refract but not on a common principal axis (Fig. 5). Three-dimensional refraction of ellipsoids is difficult to illustrate, and projections provide a better representation of true orientations. The example in Fig. 5 is from work in progress which uses the Mohr diagram for 3D strain with 'Mohr loci' (Treagus 1986) to represent interface sectional ellipses and derive graphical solutions. A consideration of the two strain components allows general rather than specific conclusions to be drawn for 3D strain refraction in the present paper.

Recall that all refracting ellipsoids share a common ellipse section on layer interfaces. The two principal axes of this ellipse (P_1, P_2), together with a third normal to layering (N), define the layer-parallel/orthogonal strain ellipsoid (component a) (Fig. 5a). The shear strain for the initial layer-normal (tan ($N \wedge N_0'$)), ($\gamma_{\perp max}$ in Treagus 1986) defines the simple shear component (b) in each layer. This component changes its value (but not direction) from layer to layer, inversely with viscosity ratio (arrowed line, Fig. 5a). The two components are mutually oblique. All three principal finite strain axes (X, Y, Z) refract in 3D (Fig. 5b) and there is therefore no common 'axis of refraction' for a multilayered system in 3D-oblique strain. As for the 2D case, layers of greater viscosity than 'standard' will exhibit finite-strain ellipsoids close to the layer-orthogonal homogeneous strain component, whereas lower viscosity layers will reflect the strong simple shear component.

The relationship of refracting XY planes to the sectional ellipse in layering is a useful measure of 3D strain refraction. In any generally oblique section of an ellipsoid, the sectional ellipse axes are not parallel to the XY plane intersection (Flinn 1962, Ramberg 1976, Borradaile 1978, Treagus & Treagus 1981, Lisle 1986, Treagus 1986). For the refracted ellipsoids in Fig. 5, it is seen that the angle between the layer elongation (P_1) and the XY plane intersection is different for the three layers. In the $V = 5$ layer, the XY plane intersects the layer very close to the layer elongation, whereas in the $V = 0.2$ layer it is more markedly oblique than for $V = 1$ (standard). This trend has been confirmed in other examples from work in progress.

End member refraction

The end members of strain refraction are the infinitely stiff layer ($V = \infty$) and the infinitely weak layer ($V = 0$).

Infinitely stiff: type 1. The layer-parallel simple shear (component b) refracts to zero (Fig. 6). Finite strain is entirely component (a), a layer-orthogonal strain. The principal axes of the strain ellipsoid will be parallel to the interface ellipse axes (P_1, P_2) and layer-normal (N) (e.g. Fig. 5b).

Infinitely weak: type 2. The layer-parallel simple shear (component b) refracts to an infinite value. Finite strain is thus infinite simple shear, together with component (a), which is negligible in comparison (Fig. 6). The X axis of the strain ellipsoid will be in the direction of the shear component (Fig. 5b, arrow), with Y at 90° in the layer plane.

In practice, layers with viscosity ratio of >10 may be regarded as type 1 with approximate layer-orthogonal strain. Likewise, layers with viscosity ratio of <0.1 will approach type 2, but with large rather than infinite layer-parallel simple shear. Regardless of the scale of simple shear, there will always be a homogeneous component of layer-orthogonal strain recorded, so that type 2 refraction is not exact plane strain.

Strain history

The finite-strain refraction patterns illustrated in Figs. 1–6 cannot provide direct information on the strain histories in each layer. Elliott (1972) pointed out that a given finite deformation can be achieved on any number of different deformation paths. (Elliott's "deformation paths" record "pure strain" and rotation histories, and are thus synonymous with 'strain history' in this paper. For a discussion of the different definitions of deformation and strain, see Means 1976, p. 150.) For example, Hobbs *et al.* (1976, p. 30) illustrated how a finite simple shear could be reached by either progressive simple shear or progressive pure shear plus rigid-body rotation.

To determine the strain history of a finite-strain state, some recorders of the strain path are required (Elliott 1972). The finite-strain ellipsoids in this paper reveal nothing of their past, but a comparison between adjacent compatible strains and consideration of the two refraction components may reveal some information on *relative strain histories*. Because of the different proportions of layer-parallel shearing in layers of different viscosity, it can be deduced that different layers will have undergone different strain histories. It might seem obvious to deduce that the progressive increase in the simple shear component with decreasing viscosity indicates a progressive increase in the non-coaxiality of strain. (For definitions of coaxial and non-coaxial strain paths, see Hsu 1966, Elliott 1972, and Means *et al.* 1980.) Such a conclusion would be convenient, and might seem intuitive, but it is not necessarily correct.

It has already been noted that simple shear can develop by pure shear and rigid rotation. Likewise, it cannot be assumed that all of the layer-parallel simple shear in the examples is progressive simple shear (implying non-coaxial straining). For example, in Fig. 1 the central standard layer may have undergone a progressive pure shear on axes parallel to the page sides, which is coaxial straining; layering would have been less inclined initially. Unstraining to maintain compatibility will require that the layers above and below had non-coaxial straining (of opposite sense). However, Fig. 3, which is identical to Fig. 1(a) but drawn with layering horizontal, prompts an interpretation which excludes layer rotation, and it would then seem obvious that the top layer was nearest to a coaxial strain history and the lower layer the most non-coaxial. This illustration serves to show that strain refraction patterns may provide useful criteria for *relative non-coaxiality* of strain paths in a multilayer, but absolute measures require that the strain path is known in one layer. If the 'bulk' deformation is specified as coaxial straining, layers of greater and lesser viscosity should be expected to have non-coaxial strain histories with opposed senses of internal vorticity (Means *et al.* 1980). However, if the most competent layers can be shown to have the closest to a coaxial strain history, as suggested by Lister & Williams (1983), the strain histories in layers of decreasing competence will be progressively more non-coaxial, in direct relationship to their layer-parallel shear component. Such a trend has important implications for the development of structures and fabrics in rocks, as outlined later. However, until it is proved correct, the trend of increasing non-coaxiality with decreasing competence should be treated only as a tentative working model.

IS CLEAVAGE REFRACTION FINITE-STRAIN REFRACTION?

The strain refraction model has demonstrated a trend with decreasing viscosity, characterized by (1) increasing strain intensity, (2) decreasing angle of the XY plane to layering, and (3) an increasingly non-coaxial strain history. In rocks, a similar trend is seen for cleavage refraction from weak approximately layer-perpendicular cleavage to intense layer-acute cleavage, from psammitic to pelitic rocks. Sorby (1853) and Harker (1886) equated such cleavage refraction to strain refraction and a greater degree of compression in the latter lithologies. Ramsay (1982) and Ramsay & Huber (1983, p. 184) follow this tradition that refracted cleavage represents finite XY-plane trajectories in rocks. However, Williams (1979) (see also Treagus 1983) questioned the evidence for this assumption in general, rather than for slates in particular (cf. Siddans 1972, Wood 1974), and asked whether exact parallelism of cleavages to principal strains could be distinguishable from subparallelism, in practice. Henderson *et al.* (1986) have recently taken up this question for cleavages which refract in folds.

Hobbs *et al.* (1976, pp. 233–246) and Williams (1976) investigated the implications for cleavage-forming processes, if cleavage planes track XY planes throughout their development. For a general non-coaxial strain history, XY planes are not material planes, but occupy progressively different material positions in the rock. The question is whether cleavages of different morphologies (see Powell 1979, Borradaile *et al.* 1982) can progressively move through a rock in the manner required (as implied by Ramsay 1982), or whether cleavages behave as material surfaces which become virtually parallel to XY planes in some situations (Williams 1976, Ghosh 1982). Strain refraction modelling will be applied to two idealized *cleavage models* in an attempt to answer this dual question.

Model 1. Cleavage refraction represents XY-plane refraction across multilayers, as maintained by Ramsay (*op. cit.*). For this model, cleavage will only represent material planes for coaxial strain histories. Cleavage refraction will exactly follow the X trajectory in Fig. 6.

Model 2. Cleavage initiated perpendicular to bedding during early layer-parallel shortening (P. F. Williams, personal communication 1982, Henderson *et al.* 1986), and was subsequently refracted during oblique straining (folding), as material planes. The cleavage angle to the layer normal (β in Treagus 1982) is thus the layer-parallel angular shear in each layer. From refraction theory, tan β ratios will give the inverse viscosity ratio (Treagus 1988). In Fig. 6, cleavage refraction would be given by the γ trajectory.

Despite the differences between the two models (and all that they imply), they appear to give rise to extremely similar theoretical orientations of cleavage for shortened layers. In Fig. 6, the X trajectory and γ trajectory are virtually indistinguishable for $V \geq 5$ and $V \leq 0.2$; in the standard ($V = 1$) layer, there is ~6° difference. Other results confirm that the XY plane and deformed layer-normal plane will be more nearly parallel with (i) increasing components of layer-parallel shortening (a), and (ii) in the most and least competent layers in the system. (This is not the case for extended layers; see Fig. 1b.)

It is known that for a large strain many material planes will move towards subparallelism to the XY plane (Ghosh 1982). This fact can account for subparallelism in the least competent layers, such that models 1 and 2 are indistinguishable, as argued by Williams (1976). In the most competent layers, if the strain history can be safely deduced as coaxial (see previous discussion), cleavages will be subnormal to bedding, subparallel to the XY plane and will be material planes. In this case, models 1 and 2 are the same. In layers of intermediate competence, two factors should control the angle between the XY plane (model 1) and the deformed layer-normal plane (model 2). With decreasing competence there is (i) increased non-coaxiality (internal vorticity; Means *et al.* 1980), moving XY planes and their temporarily coincident material planes apart, and (ii) an accompanying increased finite strain which has the compensatory effect of moving these planes together (Ghosh 1982). It seems that the intermediate layers are where the two factors are the least compensatory, so that measurable differences between cleavage models 1 and 2 exist.

A consideration of the two models in conjunction with strain refraction theory demonstrates that both models may be equally valid *empirical models* for cleavage refraction in multilayers, since the results are approximately the same for a range of competencies. Only in certain lithologies might it be possible to determine which model was more accurate. This offers a possible explanation for the dilemma posed by the apparent parallelism of cleavages to XY planes in a variety of lithologies, when a consideration of microstructural cleavage-forming processes, or evidence of shear strain on cleavage planes (Dieterich 1969, Hobbs *et al.* 1976, p. 237, Ghosh 1982) would seem to deny this to be possible.

GEOLOGICAL IMPLICATIONS OF STRAIN REFRACTION

Homogeneous strain

The theory of strain refraction predicts only two instances for homogeneous strain: in homogeneous rocks, and in layers having a history of exact layer-parallel/perpendicular principal straining. The latter case seems extremely unlikely except locally, and the former should not be expected for varied lithologies except under special conditions where the effective viscosity contrast is nil. Concepts such as 'finite homogeneous strain' and 'homogeneous flattening of folds' would thus seem to be unsound models for deformation of layered rocks of different lithologies.

Strain data

In general, strain ellipsoids (measured or computed) should vary from layer to layer in sequences of varied lithology. Variations will be in strain intensity, shape (k factor) and orientation. Strain data from a particular lithology will not be indicative of the 'bulk' strain, nor the strain in other lithologies. Thus, the prolate strains in quartzites and oblate strains in pelites described by Hutton (1979), which are characteristic near ductile shear zones in the Dalradian Caledonides (J. E. Treagus personal communication 1985), may be a feature of strain refraction. Ratschbacher & Oertel (1987) have recorded marked differences in fabric and strain for different lithologies in polyphase deformation which might also be compatible with strain refraction theory. However, their markedly prolate strains from pebbles in conglomerates could be an effect of inclusion/matrix contrast as modelled by Freeman & Lisle (1987).

Bedding–cleavage intersections

The implications of strain refraction theory to relationships of cleavage and strain have been investigated in a previous section. Strain refraction patterns have been shown to be qualitatively similar to cleavage refraction across different lithologies, regardless of whether this cleavage developed progressively parallel to XY planes, or as material surfaces which ended up nearly parallel to XY planes in varied lithologies, for the reasons already given.

For 3D strain refraction, the closeness of deformed layer-normal planes and XY planes is not yet known. Cleavages subparallel to refracted XY planes (by whatever reason) should show significant differences in bedding intersection from layer to layer. Layers of greatest competence should have cleavage–bedding intersections subparallel to the principal elongation in the layer (a potential fold axis), and thus bedding-plane stretching lineations parallel to fold hinges. In contrast, the least competent layers should show cleavage–bedding intersections markedly oblique to fold hinges and bedding-plane stretching lineations, and potential fold hinges transected by cleavage. Stringer & Treagus (1980) noted differences in cleavage–bedding intersection and fold transection for different lithologies in the Southern Uplands of Britain comparable to those described here. However, 3D oblique strain is only one of many possible explanations for the transection of folds by a contemporaneous cleavage (compare Powell 1974, Borradaile 1978, Stringer & Treagus 1980, Gray 1981, Treagus & Treagus 1981, Soper & Hutton 1984, Soper 1986).

Heterogeneous simple shear, shear criteria and strain history

Strain refraction theory suggests that all layers in a sequence of contrasting lithologies can be modelled as layer-width uniform shear zones with simple shear proportional to viscosity, together with a 'regional' layer-parallel/orthogonal homogeneous strain. Thus, for decreasing viscosity there is increasing strain intensity, which goes hand-in-hand with increasingly non-coaxial straining.

Such a rule has important implications for strain histories and shear-strain criteria in rocks. Within a set of parallel layers of contrasting lithology, the sense of shear should be the same, but the values very different, ranging from nearly zero in the most competent to a maximum in the least competent. Such differences would be reflected in changes in "structural symmetry" (Choukroune *et al.* 1987) and differences in microstructural shear criteria (Simpson & Schmid 1983) across different lithologies. Competent layers will most probably exhibit bedding-symmetrical fabrics and structures characteristic of an approximate coaxial strain history, whereas incompetent layers should exhibit non-coaxial asymmetric fabrics and structures (Williams 1979, Lister & Williams 1983). Where asymmetry of crystallographic fabrics is used to infer the sense of shear (Lister & Williams 1983), competent layers should exhibit subsymmetric fabrics, and incompetent layers strongly asymmetric fabrics.

Fabric analyses in thrust zones

Analyses of quartz c-axis fabrics close to the Moine Thrust in NW Scotland (Law 1987, Law & Potts 1987) have demonstrated the presence of two components of strain: a homogeneous coaxial component (symmetric fabric) and heterogeneous (asymmetric) component. Law and Potts interpret the former as a homogeneous pure strain (a regional 'flattening'), and the latter a heterogeneous simple shear of increasing intensity towards the thrust plane. Even where the fabric is strongly asymmetric near the thrust, the component of flattening can still be distinguished (R. D. Law, personal communication 1987). The results of Law and Potts have remarkable similarities with strain refraction modelling; their quartzites have behaved as if there was a progressive decrease of viscosity approaching the thrust, analogous to Ramsay's "deformation-induced competence contrast" in shear zones (1982).

Fracture refraction

The strain refraction theorem presented here is for viscous layers and cannot therefore model fracture. However, Foster & Hudleston (1986) adapted the viscous stress refraction equation (Treagus 1973) to elastic behaviour, in order to interpret refracted fracture cleavage. The same principles might be applied to joints which refract (Hancock 1985). Stress refraction theory predicts stresses subparallel and subperpendicular to bedding in elastically competent lithologies, which could explain the common occurrence of bedding perpendicular joints and rectangular boudins in lithologies such as sandstone and limestone. Tensile fractures in very incompetent lithologies, if they formed, would be close to 45° to the bedding.

Although theory predicts the refraction of principal stresses from layer to layer, it cannot be assumed that faults would necessarily reflect such refraction. Faults will develop and propagate from an initial crack, which it seems reasonable to suppose is in the most competent/brittle unit. The propagation of cracks, their linkage into larger fractures and subsequent finite displacements as faults are beyond the scope of this paper. However, it is tentatively suggested that the ramp-flat arrangement of thrust faults (Rich 1934) might be a manifestation of stress and strain refraction through contrasting lithologies. Ramps are characteristic of competent lithologies, and could be where thrusts initiate (Eisenstadt & De Paor 1987). Flats subparallel to bedding in particular lithologies may reflect strong fracture refraction, or alternatively, they may be layer-parallel shear zones developed in highly incompetent layers.

Just as strain data from individual lithologies will not indicate the bulk or regional strain, joint and fault data from one rock type (usually competent) should not be expected to indicate the regional stress orientations.

CONCLUSIONS

The theory of strain refraction across lithological contrasts has many similarities with other models of heterogeneous strain. However, unlike models dependent on geometrical compatibility alone, the refraction model depends on both mechanical and geometrical compatibility. Strain ellipsoids are considered to refract because of changes in the bedding-parallel shear strain component. Across competence contrasts, the bedding-parallel shear strain ratio is the inverse of the effective viscosity ratio.

The geological implications of strain refraction are as follows.

(1) Strain will not be homogeneous across varied lithologies, except very locally.

(2) Refracting strains in a multilayer can be factorized into (a) a homogeneous layer-parallel/perpendicular (called layer-orthogonal) strain, and (b) a variable layer-parallel simple shear, which are contemporaneous. All layers show the same sense of shear, but the amount will be inversely proportional to competence.

(3) The simplest strain refraction is for $Y = 1$ parallel to layering. All ellipsoids will refract on the Y axis, strain variations are in 2D and all ellipsoids are $k = 1$.

(4) A more general 2D strain refraction occurs when $Y \neq 1$, or X, or Z are parallel to layering. Strong ellipsoid shape variations occur with refraction, and in some cases there will be a change-over of principal axes. In general, least competent layers will be closest to plane-strain ($k = 1$) ellipsoids. Different rock types should be expected to exhibit strain ellipsoids of different shape, intensity and orientation, and show cleavage refraction.

(5) True 3D strain refraction involves a refraction of all three ellipsoid axes, and associated shape changes. The 3D refraction of XY planes means that they will not have parallel bedding intersections for different rock types. For cleavages subparallel to XY planes, cleavage–bedding intersections should be different for different lithologies: subparallel to potential fold hinges in competent rocks, and oblique (i.e. transecting) in incompetent. Thus, cleavage may transect folds in incompetent lithologies but fan symmetrically around folds in competent layers.

(6) Very competent rock types will approach the homogeneous layer-orthogonal strain component, subcoaxial with bedding. Kinematic indicators may reveal an approximate coaxial strain history, but one where principal directions may change in relative value. Layers in compression should develop a weak cleavage subperpendicular to bedding (i.e. subparallel to the XY plane).

(7) Intermediate competencies with modest components of shear may be most useful for investigating whether cleavage forms exactly parallel to XY planes of strain, represents the material initial layer-normal plane, or neither.

(8) The least competent lithologies will develop the most extreme strain values, and appear dominated by layer-parallel simple shear. Structures and fabrics will be strongly asymmetric, indicating the non-coaxial strain history. Cleavages are predicted subparallel to XY planes, regardless of whether they represent exact non-material XY planes or are deformed material surfaces.

(9) Cleavage/bedding angles may provide empirical data on effective viscosity ratios in rocks.

(10) Fractures should reveal features of stress and strain refraction. Joints normal to bedding, and rectilinear boudins may be evidence of refracted stresses in competent/brittle rocks. Ramp-flat geometry may be the result of fracture refraction across marked lithological contrasts.

Acknowledgements—I am grateful for a University of Manchester Honorary Research Fellowship which allowed me the facilities of the Geology Department for research. I appreciate the suggestions for improvement made by Paul Williams, Graham Borradaile, Peter Hudleston and Win Means, and the encouragement and forebearance of Jack Treagus. Richard Hartley is thanked for drawing the figures.

REFERENCES

Biot, M. A. 1965. *Mechanics of Incremental Deformations*. John Wiley & Sons, New York.

Borradaile, G. J. 1978. Transected folds: a study illustrated with examples from Canada and Scotland. *Bull. geol. Soc. Am.* **89**, 481–493.

Borradaile, G. J., Bayly, M. B. & Powell, C. McA. (editors) 1982. *Atlas of Deformational and Metamorphic Rock Fabrics*. Springer Verlag, New York.

Choukroune, P., Gapais, D. & Merle, O. 1987. Shear criteria and structural symmetry. *J. Struct. Geol.* **9**, 525–530.

Cobbold, P. R. 1977. Description and origin of banded deformation structures. I. Regional strain, local perturbations and deformation bands. *Can. J. Earth Sci.* **14**, 1721–1731.

Cobbold, P. R. 1983. Kinematic and mechanical continuity at a coherent interface. *J. Struct. Geol.* **5**, 341–349.

Cobbold, P. R. & Barbotin, E. 1988. The geometric significance of strain trajectory curvature. *J. Struct. Geol.* **10**, 211–218.

Cobbold, P. R., Cosgrove, J. W. & Summers, J. M. 1971. Development of internal structures in deformed anisotropic rocks. *Tectonophysics* **12**, 25–53.

Coward, M. P. & Kim, J. H. 1981. Strain within thrust sheets. In: *Thrust and Nappe Tectonics* (edited by McClay, K. & Price, N. J.). *Spec. Publs geol. Soc. Lond.* **9**, 275–292.

Cutler, J. M. & Cobbold, P. R. 1985. A geometric approach to two-dimensional finite strain compatibility. *J. Struct. Geol.* **7**, 727–735.

Dieterich, J. H. 1969. Origin of cleavage in folded rocks. *Am. J. Sci.* **267**, 155–165.

Eisenstadt, G. & De Paor, D. G. 1987. Alternative model of thrust-fault propagation. *Geology* **15**, 630–633.

Elliott, D. 1972. Deformation paths in structural geology. *Bull. geol. Soc. Am.* **83**, 2621–2638.

Flinn, D. 1962. On folding during three-dimensional progressive deformation. *Q. Jl geol. Soc. Lond.* **118**, 385–433.

Foster, M. E. & Hudleston, P. J. 1986. "Fracture cleavage" in the Duluth Complex, northeastern Minnesota. *Bull. geol. Soc. Am.* **97**, 85–96.

Freeman, B. & Lisle, R. J. 1987. The relationship between tectonic strain and the three-dimensional shape fabrics of pebbles in deformed conglomerates. *J. geol. Soc. Lond.* **144**, 635–639.

Ghosh, S. K. 1982. The problem of shearing along axial plane foliations. *J. Struct. Geol.* **4**, 63–67.

Gray, D. R. 1981. Cleavage–fold relationships and their implications for transected folds: an example from Southwest Virginia, U.S.A. *J. Struct. Geol.* **3**, 265–277.

Hancock, P. L. 1985. Brittle microtectonics: principles and practice. *J. Struct. Geol.* **7**, 437–457.

Hara, I. & Shimamoto, T. 1984. Folds and folding. In: *Geological Structures* (edited by Uemura, T. & Mizutani, S.). John Wiley & Sons, London, 191–244.

Harker, A. 1886. On slaty cleavage and allied rock structures with special reference to the mechanical theories of their origin. *Rep. Br. Ass. Adv. Sci.* (1885), 813–852.

Henderson, J. R., Wright, T. O. & Henderson, M. N. 1986. A history of cleavage and folding: An example from the Goldenville Formation, Nova Scotia. *Bull. geol. Soc. Am.* **97**, 1354–1366.

Hobbs, B. E., Means, W. D. & Williams, P. F. 1976. *An Outline of Structural Geology*. John Wiley & Sons, New York.

Hsu, T. C. 1966. The characteristics of coaxial and non-coaxial strain paths. *J. Strain Analysis* **1**, 216–222.

Hutton, D. H. W. 1979. Tectonic slides: a review and reappraisal. *Earth Sci. Rev.* **15**, 151–172.

Law, R. D. 1987. Heterogeneous deformation and quartz crystallographic fabric transitions: natural examples from the Moine Thrust zone at the Stack of Glencoul, northern Assynt. *J. Struct. Geol.* **9**, 819–833.

Law, R. D. & Potts, G. J. 1987. The Tarskavaig Nappe of Skye, northwest Scotland: a re-examination of the fabrics and their kinematic significance. *Geol. Mag.* **124**, 231–248.

Lisle, R. J. 1986. The sectional strain ellipse during progressive coaxial deformations. *J. Struct. Geol.* **8**, 809–817.

Lister, G. S. & Williams, P. F. 1983. The partitioning of deformation in flowing rock masses. *Tectonophysics* **92**, 1–33.

Means, W. D. 1976. *Stress and Strain*. Springer Verlag, New York.

Means, W. D. 1983. Application of the Mohr-circle construction to problems of inhomogeneous deformation. *J. Struct. Geol.* **5**, 279–286.

Means, W. D., Hobbs, B. E., Lister, G. S. & Williams, P. F. 1980. Vorticity and non-coaxiality in progressive deformations. *J. Struct. Geol.* **2**, 371–378.

Powell, C. McA. 1974. The timing of slaty cleavage during folding of Precambrian rocks, Northwest Tasmania. *Bull. geol. Soc. Am.* **85**, 1043–1060.

Powell, C. McA. 1979. A morphological classification of rock cleavage. *Tectonophysics* **58**, 21–34.

Ramberg, H. 1964. Selective buckling of composite layers with contrasting rheological properties, a theory for simultaneous formation of several orders of folds. *Tectonophysics* **1**, 307–341.

Ramberg, H. 1976. The strain in a sheet intersecting the strain ellipsoid at any angle. *Bull. Soc. geol. Fr.* 7 *Ser.* **18**, 1417–1422.

Ramsay, J. G. 1967. *Folding and Fracturing of Rocks*. McGraw-Hill, New York.

Ramsay, J. G. 1980. Shear zone geometry: a review. *J. Struct. Geol.* **2**, 83–100.

Ramsay, J. G. 1982. Rock ductility and its influence on the development of tectonic structures in mountain belts. In: *Mountain Building Processes* (edited by Hsü, K. J.). Academic Press, London, 111–127.

Ramsay, J. G. & Graham, R. H. 1970. Strain variations in shear belts. *Can. J. Earth Sci.* **7**, 786–813.

Ramsay, J. G. & Huber, M. I. 1983. *The Techniques of Modern Structural Geology. Volume* 1: *Strain Analysis*. Academic Press, London.

Ramsay, J. G. & Huber, M. I. 1987. *The Techniques of Modern Structural Geology. Volume* 2: *Folds and Fractures*. Academic Press, London.

Ratschbacher, L. & Oertel, G. 1987. Superposed deformations in the Eastern Alps: strain analysis and microfabrics. *J. Struct. Geol.* **9**, 263–276.

Rich, J. L. 1934. Mechanics of low-angle overthrust faulting as illustrated by Cumberland thrust block, Virginia, Kentucky and Tennessee. *Bull. Am. Ass. Petrol. Geol.* **18**, 1584–1587.

Roberts, D. 1971. Abnormal cleavage patterns in fold hinge zones from Varanger peninsula, Northern Norway. *Am. J. Sci.* **271**, 170–180.

Sanderson, D. J. 1982. Models of strain variations in nappes and thrust sheets: a review. *Tectonophysics* **88**, 201–233.

Shimamoto, T. & Hara, I. 1976. Geometry and strain distribution of single-layer folds. *Tectonophysics* **30**, 1–34.

Siddans, A. W. B. 1972. Slaty cleavage—a review of research since 1815. *Earth Sci. Rev.* **8**, 205–232.

Simpson, C. & Schmid, S. M. 1983. An evaluation of criteria to deduce the sense of movement in sheared rocks. *Bull. geol. Soc. Am.* **94**, 1281–1288.

Soper, N. J. 1986. Geometry of transecting, anastamosing solution cleavage in transpression zones. *J. Struct. Geol.* **8**, 937–940.

Soper, N. J. & Hutton, D. H. W. 1984. Late Caledonian sinistral displacements in Britain: implications for a three-plate collision model. *Tectonics* **3**, 781–794.

Sorby, H. C. 1853. On the origin of slaty cleavage. *Edinb. New Philos. J.* **55**, 137–148.

Strömgård, K.-E. 1973. Stress distribution during formation of boudinages and pressure shadows. *Tectonophysics* **16**, 215–248.

Stringer, P. & Treagus, J. E. 1980. Non-axial planar S_1 cleavage in the Hawick Rocks of the Galloway area, Southern Uplands, Scotland. *J. Struct. Geol.* **2**, 317–331.

Treagus, J. E. & Treagus, S. H. 1981. Folds and the strain ellipsoid: a general model. *J. Struct. Geol.* **3**, 1–17.

Treagus, S. H. 1973. Buckling stability of a viscous single-layer system oblique to the principal compression. *Tectonophysics* **19**, 271–289.

Treagus, S. H. 1981. A theory of stress and strain variations in viscous layers, and its geological implications. *Tectonophysics* **72**, 75–103.

Treagus, S. H. 1982. A new isogon-cleavage classification and its application to natural and model fold studies. *Geol. J.* **17**, 49–64.

Treagus, S. H. 1983. A theory of finite strain variation through contrasting layers, and its bearing on cleavage refraction. *J. Struct. Geol.* **5**, 351–368.

Treagus, S. H. 1986. Some applications of the Mohr diagram for three-dimensional strain. *J. Struct. Geol.* **8**, 819–830.

Treagus, S. H. 1988. A history of cleavage and folding: an example from the Goldenville Formation, Nova Scotia: Discussion. *Bull. geol. Soc. Am.* **99**, 152–153.

Williams, P. F. 1976. Relationships between axial-plane foliations and strain. *Tectonophysics* **30**, 181–196.

Williams, P. F. 1979. The development of asymmetrical folds in a cross-laminated siltstone. *J. Struct. Geol.* **1**, 19–30.

Wood, D. S. 1974. Current views on the development of slaty cleavage. *A. Rev. Earth Planet. Sci.* **2**, 1–37.